Some topological descriptors and algebraic polynomials of $P_m+F_Pm$

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A topological index of $G$ is a quantity related to $G$ that characterizes its topology. Properties of the chemical compounds and topological invariants are related to each other. In this paper, we derive the algebraic polynomials including first and second Zagreb polynomials, and forgotten polynomial for $P_m+F_Pm$. Further, we worked on the hyper-Zagreb, first and second multiple Zagreb indices, and forgotten index of these graphs. Consider the molecular graph with atoms to be taken as vertices and bonds can be shown by edges. For such graphs, we can determine the topological descriptors showing their bioactivity as well as their physiochemical characteristics. Moreover, we derive graphical representation of our outcomes, depicting the technical dependence of topological indices and polynomials on the involved structural parameters.

KEYWORDS
Algebraic polynomial; topological descriptor; Zagreb indices.

Introduction

In chemical graph theory, and Mathematics Chemistry, a topological descriptor is a sort of a molecular topological invariant that is computed based on the molecular structure of a chemical compound. A big quantity of chemical experiments needs a resolution of the chemical characteristics of compounds and drugs. The chemical-based experiments demonstrate that there is strong inherent correlation between the chemical characteristics of chemical compounds and drugs, and molecular structures. Topological invariants deliberated for the chemical structures can be helpful for us to work on the physical features, chemical reactivity, and biological activity.

A topological index is designed by reforming a chemical structure into a quantity. These topological invariants are associated with some physicochemical characteristics like stability, boiling point, strain energy etc. of chemical compounds. These are computed by the help of their definitions. Chemical reaction network theory is a field of applied mathematics that is beneficial to model the changing in real world chemical systems. Since its beginning in the 1960s, it has attracted the attraction of the researchers in developing research areas, only because of its importance in biochemistry and theoretical Chem. It has also drawn the interest among pure mathematicians because of its interesting problems that come to light from the Mathematics patterns in structures of material.

Shirdel et al. [1] worked on the hyper Zagreb index and gave its mathematics representation as follows:

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2$$

(1)

Ghorbani and Azimi [2] worked on two new variations of Zagreb indices; first multiple Zagreb index.
\[ PM_1(G) = \prod_{uv \in E(G)} (d_u + d_v) \]  

and second multiple Zagreb index
\[ PM_2(G) = \prod_{uv \in E(G)} (d_u \times d_v) \]  

Furtula and Gutman [3] proposed a very beneficial topological index known as the forgotten index and described as:
\[ F(G) = \sum_{uv \in E(G)} (d_u^2 + d_v^2) \]  

The first Zagreb polynomial of \( G \) is defined in [4, 5] as follows:
\[ M_1(G, x) = \sum_{uv \in E(G)} x^{d_u + d_v} \]  

The second Zagreb polynomial of \( G \) is defined in [4, 5] as follows:
\[ M_2(G, x) = \sum_{uv \in E(G)} x^{d_u \times d_v} \]  

Main results
In this paper we computed the first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial of \( P_m + Q P_m \). We also computed some degree-based topological indices such as first multiple Zagreb index, second multiple Zagreb index, Hyper Zagreb index and forgotten index or F-index of these networks. Further information on topological indices are available in the literature [6-24].

\textbf{TABLE 1} Edge partition of \( P_m + Q P_m \) based on degree of end vertices of each edge

<table>
<thead>
<tr>
<th>(d_u, d_v)</th>
<th>[2; 3)</th>
<th>(3; 3]</th>
<th>(3; 4]</th>
<th>(4; 4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>8</td>
<td>6+4t</td>
<td>10(t-1)t</td>
<td>4t+5t-3</td>
</tr>
</tbody>
</table>

\textbf{Q-Operation for \( P_m \)}

The graph \( P_m + Q P_m \) have \( 2t^2+7t+6 \) vertices and \( 4t^2+19t+21 \) edges (Table 1).

\textbf{Theorem 1.} Let \( G \equiv P_m + Q P_m \) be the graph then the first Zagreb polynomial for this graph is:
\[ M_1(P_m + Q P_m, x) = 8x^5 + (6+4t)x^6 + 10(t+1)x^7 + (4t^2 + 5t - 3)x^8 \]

\textbf{Proof.} By definition of first Zagreb polynomial

\[ M_1(G, x) = \sum_{uv \in E(G)} x^{(d_u + d_v)} = M_1(P_m + Q P_m, x) = \sum_{d_u, d_v \in E(G)} x^{d_u + d_v} \]

\[ + \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v} \]

\[ = |E_1(P_m + Q P_m)| x^{(2+3)} + |E_2(P_m + Q P_m)| x^{(3+3)} \]

\[ + |E_3(P_m + Q P_m)| x^{(3+4)} + |E_4(P_m + Q P_m)| x^{(4+4)} \]

\[ = 8x^5 + (6+4t)x^6 + 10(t+1)x^7 + (4t^2 + 5t - 3)x^8 \]

\textbf{Theorem 2.} Let \( G \equiv P_m + Q P_m \) be the graph then the first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial for this graph is:
\[ M_2(P_m + Q P_m, x) = 8x^6 + (6+4t)x^7 + 10(t+1)x^{12} + (4t^2 + 5t - 3)x^{16} \]

\textbf{Proof.} By definition of second Zagreb polynomial we have,
\[ M_2(G, x) = \sum_{uv \in E(G)} x^{d_u \times d_v} \]

\[ + \sum_{u \in E(P_m \cup P_m)} x^{d_u \times d_v} \]

\[ = |E_1(P_m + Q P_m)| x^{(2+3)} |E_2(P_m + Q P_m)| x^{(3+3)} \]

\[ + |E_3(P_m + Q P_m)| x^{(3+4)} + |E_4(P_m + Q P_m)| x^{(4+4)} \]

\[ = 8x^6 + (6+4t)x^7 + 10(t+1)x^{12} + (4t^2 + 5t - 3)x^{16} \]

\textbf{Theorem 3.} Let \( G \equiv P_m + Q P_m \) be the graph then the first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial for this graph is:
\[ F(P_m + Q P_m, x) = 8x^{13} + (6+4t)x^{18} + 10(t+1)x^{25} + (4t^2 + 5t - 3)x^{32} \]

\textbf{Proof.} By definition of forgotten polynomial, we have:
\[ F(G, x) = \sum_{uv \in E(G)} x^{(d_u + d_v)} = F(P_m + Q P_m, x) \]

\[ = \sum_{u \in E(P_m \cup P_m)} x^{(d_u + d_v)} \]

\[ + \sum_{u \in E(P_m \cup P_m)} x^{(d_u + d_v)} \]

\[ = |E_1(P_m + Q P_m)| x^{(2+3)} |E_2(P_m + Q P_m)| x^{(3+3)} \]

\[ + |E_3(P_m + Q P_m)| x^{(3+4)} + |E_4(P_m + Q P_m)| x^{(4+4)} \]

\[ = 8x^{13} + (6+4t)x^{18} + 10(t+1)x^{25} + (4t^2 + 5t - 3)x^{32} \]

\textbf{Example:} Graphs of first Zagreb polynomial, second Zagreb polynomial and forgot-ten polynomial are shown in Figure 1.

\textbf{Preposition:} Let \( G \equiv P_m + Q P_m \) be the graph then the hyper Zagreb index, first multiple Zagreb
index, second multiple Zagreb index and forgotten index are:

\[ \text{HM}(P_m + Q P_m) = 200 + 36(6 + 4t) + 490(t + 1) + 64(4t^2 + 5t - 3); \]
\[ \text{PM}_1(P_m + Q P_m) = 5^8 + 6^8 + 14t^{10} + 8^4t^4 + 5t^2 - 3; \]
\[ \text{PM}_2(P_m + Q P_m) = 6^8 + 9^6t^4 + 12^10t^{10} + 16^4t^4 + 5t^2 - 3; \]
\[ F(P_m + Q P_m) = 13^8 + 25^6t^4 + 71^0t^{10} + 32^4t^4 + 5t^2 - 3. \]

**FIGURE 1** Graph of Algebraic polynomials for \((P_m+Q P_m)\)

**FIGURE 2** Graph of topological indices for \((P_m+Q P_m)\)

Proof. (1). Let \(G \supseteq P_m + Q P_m\) be the graph then by equation (1).

\[ \text{HM}(G) = \sum_{uv \in E(G)} (d_u + d_v)^2; \]
\[ \text{HM}(P_m + Q P_m) = \sum_{uv \in E(P_m + Q P_m)} (d_u + d_v)^2 + \sum_{uv \in E(P_m + Q P_m)} (d_u + d_v)^2; \]
\[ \text{HM}(P_m + Q P_m) = \sum_{uv \in E(P_m + Q P_m)} (d_u + d_v)^2 + \sum_{uv \in E(P_m + Q P_m)} (d_u + d_v)^2; \]
\[ \text{HM}(P_m + Q P_m) = 200 + 36(6 + 4t) + 490(t + 1) + 64(4t^2 + 5t - 3). \]

(2). by definition

\[ \text{PM}_1(G) = \prod_{uv \in E(G)} (d_u + d_v) \]
\[ \text{PM}_1(P_m + Q P_m) = \prod_{uv \in E(P_m + Q P_m)} (d_u + d_v) \]
\[ \text{PM}_1(P_m + Q P_m) = \prod_{uv \in E(P_m + Q P_m)} (d_u + d_v); \]
\[ \text{PM}_2(P_m + Q P_m) = (5)^8 + (6)^6t^4 + (7)^10t^{10} + (8)^4t^4 + 5t^2 - 3. \]

(3). by definition

\[ \text{PM}_2(P_m + Q P_m) = \prod_{uv \in E(P_m + Q P_m)} (d_u + d_v) \]
\[ \text{PM}_2(P_m + Q P_m) = \prod_{uv \in E(P_m + Q P_m)} (d_u + d_v); \]
\[ \text{PM}_2(P_m + Q P_m) = (5)^8 + (6)^6t^4 + (7)^10t^{10} + (8)^4t^4 + 5t^2 - 3. \]

3D plot for hyper Zagreb index, first Zagreb index and forgotten index are shown in Figure 2.

R-Operation for \(P_m\)

For graph \(P_m + R P_m\) the cardinality of vertex and edge sets are \(2t^2 + 7t + 6\) and \(4(t+1)(t+2)\), respectively.
### Table 2: Edge partition of \( G \otimes P_m + r \) based on degree of end vertices of each edge

<table>
<thead>
<tr>
<th>(d, d)</th>
<th>(2; 3)</th>
<th>(2; 4)</th>
<th>(2; 5)</th>
<th>(2; 6)</th>
<th>(3; 4)</th>
<th>(3; 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>4</td>
<td>2t</td>
<td>4t</td>
<td>2t²</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>(d, d)</td>
<td>(4; 4)</td>
<td>(4; 6)</td>
<td>(5; 5)</td>
<td>(5; 6)</td>
<td>(6; 6)</td>
<td>.</td>
</tr>
<tr>
<td>Frequency</td>
<td>2t²</td>
<td>2t</td>
<td>2t</td>
<td>2(t-1)</td>
<td>2t²</td>
<td>.</td>
</tr>
</tbody>
</table>

**Theorem 4.** Let \( G \otimes P_m + r \) be the graph then the first Zagreb polynomial for this graph is

\[
M_1(G, x) = \sum_{x \in V(G)} x^{d(x, d(x))} + \sum_{u \in E(G)} x^{d(u, u)}
\]

**Proof.** By definition of first Zagreb polynomial we have:

\[
M_1(P_m + r, x) = \sum_{x \in V(P_m + r)} x^{d(x, d(x))} + \sum_{u \in E(P_m + r)} x^{d(u, u)}
\]

**Theorem 5.** Let \( G \otimes P_m + r \) be the graph then the second Zagreb polynomial for this graph is

\[
M_2(G, x) = \sum_{x \in V(G)} x^{d(x, d(x))} + \sum_{u \in E(G)} x^{d(u, u)}
\]

**Proof.** By definition of second Zagreb polynomial, we have:

\[
M_2(P_m + r, x) = \sum_{x \in V(P_m + r)} x^{d(x, d(x))} + \sum_{u \in E(P_m + r)} x^{d(u, u)}
\]

**Theorem 6.** Let \( G \otimes P_m + r \) be the graph then the forgotten polynomial for this graph is

\[
F(P_m + r, x) = x^{13} + 2x^{20} + 4x^{29} + 2x^{30} + 4x^{35} + 4x^{34} + (2x^2 - 2x^2)^2 + (2x^2 - 2x^2)^3 + (2x^2 - 2x^2)^4
\]

**Preposition:** Let \( G \otimes P_m + r \) be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are:

\[
H(G) = \sum_{x \in V(G)} d(x, d(x)) + \sum_{u \in E(G)} d(u, u)
\]

\[
M_1(G, x) = \sum_{x \in V(G)} x^{d(x, d(x))} + \sum_{u \in E(G)} x^{d(u, u)}
\]

\[
M_2(G, x) = \sum_{x \in V(G)} x^{d(x, d(x))} + \sum_{u \in E(G)} x^{d(u, u)}
\]

\[
F(G, x) = x^{13} + 2x^{20} + 4x^{29} + 2x^{30} + 4x^{35} + 4x^{34} + (2x^2 - 2x^2)^2 + (2x^2 - 2x^2)^3 + (2x^2 - 2x^2)^4
\]

\[
H(G) = \sum_{x \in V(G)} d(x, d(x)) + \sum_{u \in E(G)} d(u, u)
\]

\[
M_1(G, x) = \sum_{x \in V(G)} x^{d(x, d(x))} + \sum_{u \in E(G)} x^{d(u, u)}
\]

\[
M_2(G, x) = \sum_{x \in V(G)} x^{d(x, d(x))} + \sum_{u \in E(G)} x^{d(u, u)}
\]

\[
F(G, x) = x^{13} + 2x^{20} + 4x^{29} + 2x^{30} + 4x^{35} + 4x^{34} + (2x^2 - 2x^2)^2 + (2x^2 - 2x^2)^3 + (2x^2 - 2x^2)^4
\]
Proof. (1). Let $G \cong P_m + P_m$ be the graph then by equation (1), we have:

$$HM(G) = \sum_{(u,v) \in E(G)} (d_u + d_v)^2$$

$$= \sum_{uv \notin E(P_m \cup P_m)} (d_u + d_v)^2 + \sum_{uv \in E(P_m \cup P_m)} (d_u + d_v)^2$$

$$= \sum_{uv \notin E(P_m \cup P_m)} (d_u + d_v)^2 + \sum_{uv \in E(P_m \cup P_m)} (d_u + d_v)^2$$

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3D plot for hyper Zagreb index, first Zagreb index, second Zagreb index and forgotten index are shown in Figure 4.

$S$-Operation for $P_m$

The graph $P_m \ast P_m$ have order $2t^2 + 7t + 6$ and size $3(t+1)(t+2)$.

Theorem 7. Let $G \cong P_m + P_m$ be the graph then the first Zagreb polynomial for this graph is

$$M_1(P_m + g P_m, x) = 4x^4 + (4 + 6)x^4 + (22 - 2) x^5 + 2x^6 + x(t - 1)x^8.$$

Proof. By definition of first Zagreb polynomial, we have:

$$M_1(G, x) = \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

$$= \sum_{uv \notin E(P_m \cup P_m)} x^{d_u + d_v} + \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

$$= \sum_{uv \notin E(P_m \cup P_m)} x^{d_u + d_v} + \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

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$$= \sum_{uv \notin E(P_m \cup P_m)} x^{d_u + d_v} + \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

Two graphs $P_m \ast P_m$, $P_m \ast P_m$ and $P_m \ast P_m$ are shown in Figure 4.

Theorem 8. Let $G \cong P_m + P_m$ be the graph then the second Zagreb polynomial for this graph is

$$M_2(P_m + g P_m, x) = 4x^4 + (4 + 6)x^5 + (2^2 - 2) x^6 + 2x^7 + x(t - 1)x^8.$$

Proof. By definition of second Zagreb polynomial, we have:

$$M_2(G, x) = \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

$$= \sum_{uv \notin E(P_m \cup P_m)} x^{d_u + d_v} + \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

$$= \sum_{uv \notin E(P_m \cup P_m)} x^{d_u + d_v} + \sum_{uv \in E(P_m \cup P_m)} x^{d_u + d_v}$$

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Two graphs $P_m \ast P_m$, $P_m \ast P_m$ and $P_m \ast P_m$ are shown in Figure 4.
TABLE 3 Edge partition of $P_m+sP_m$ based on degree of end vertices of each edge.

<table>
<thead>
<tr>
<th>(d, d)</th>
<th>(2, 2)</th>
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<tr>
<td>Frequency</td>
<td>4</td>
<td>6+4t</td>
<td>2t-2</td>
<td>2t</td>
<td>t(1-t)</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 3 Graph of Algebraic polynomials for $P_m+sP_m$

FIGURE 4 Graph of topological indices for $(P_m+sP_m)$

**Theorem 9.** Let $G\cong P_m+sP_m$ be the graph then the forgotten polynomial for this graph is

$$F(P_m+sP_m, x) = 4x^3 + (4+6)x^4 + (2+7)x^5 + (2-2)x^6 + 2x^7 + (t(1-t))x^8.$$  

**Proof.** By definition of forgotten polynomial

$$F(G, x) = \sum_{a \in V(G)} x^{d(a)}.$$  

**Theorem 9.** Let $G\cong P_m+sP_m$ be the graph then the forgotten polynomial for this graph is

$$F(P_m+sP_m, x) = 4x^3 + (4+6)x^4 + (2+7)x^5 + (2-2)x^6 + 2x^7 + (t(1-t))x^8.$$  

The graph of first Zagreb, second Zagreb and Forgotten polynomials for $P_m+sP_m$ are shown in Figure 5 below.

**Proposition:** Let $G\cong P_m+sP_m$ be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are

$$HM(P_m+sP_m) = 64 + 25(4 + 6) + 36(2 + 2) + 36(2 - 2) + (49)2 + 64(t - 1);$$  

$$PM_1(P_m+sP_m) = 4 + 4 + 4 + 2 + 2 + 2 + 2 + 122 + 1 + 16(t - 1);$$  

$$PM_2(P_m+sP_m) = 4 + 6 + 6 + 6 + 6 + 6 + 9 + 22 = 122 + 1 + 16(t - 1);$$  

$$F(P_m+sP_m) = 4^3 + 13 + 122 + 22 + 22 = 25 + 32 + 32.$$  

**Proof (1).** Let $G\cong P_m+sP_m$ be the graph then by equation (1).

$$HM(P_m+sP_m) = \sum_{a \in V(G)} (d_a + d_e)^2 + \sum_{u \in E(G)} (d_u + d_v)^2$$  

$$+ \sum_{u \in E(G)} (d_u + d_v)^2 + \sum_{u \in E(G)} (d_u + d_v)^2$$  

$$+ \sum_{u \in E(G)} (d_u + d_v)^2 + \sum_{u \in E(G)} (d_u + d_v)^2$$  

$$= 64 + 25(4 + 6) + 36(2 + 2) + 36(2 - 2) + (49)2 + 64(t - 1).$$

(2). By definition
Some topological descriptors algebraic ...
Theorem 10. Let \( G \cong P_m + T_m \) be the graph then the first Zagreb polynomial for this graph is
\[
M_1(P_m + T_m) = 4x^4 + (8 + 4x)^2 + 8x^8 + 2x^2 + (2^2 + 6)x^8 + 40 - 1)x^3 + 2(2x + 2t - 1)x^3 + 2t - 1)x^2.
\]

**Proof.** By definition of first Zagreb polynomial, we have:
\[
M_1(G, x) = \sum_{v \in V(G)} \sum_{1 \leq d \leq k} x^{d deg(v)} = M_1(P_m + T_m, x) = \sum_{v \in V(G)} x^{deg(v)}
\]
\[
+ \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
+ \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
+ \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
|E(P_m + T_m)| = 4x^4 + (8 + 4x)^2 + 8x^8 + 2x^2 + (2^2 + 6)x^8 + 40 - 1)x^3 + 2(2x + 2t - 1)x^3 + 2t - 1)x^2.
\]

Theorem 11. Let \( G \cong P_m + P_m \) be the graph then the second Zagreb polynomial for this graph is
\[
M_2(P_m + P_m, x) = 4x^4 + (8 + 4x)^2 + 8x^8 + 2x^2 + (2^2 + 6)x^8 + 40 - 1)x^3 + 2(2x + 2t - 1)x^3 + 2t - 1)x^2.
\]

**Proof.** By definition of second Zagreb polynomial, we have:
\[
M_2(G, x) = \sum_{v \in V(G)} x^{deg(v)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
+ \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
+ \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
M_2(P_m + P_m, x) = 4x^4 + (8 + 4x)^2 + 8x^8 + 2x^2 + (2^2 + 6)x^8 + 40 - 1)x^3 + 2(2x + 2t - 1)x^3 + 2t - 1)x^2.
\]

Theorem 12. Let \( G \cong P_m + P_m \) be the graph then the forgotten polynomial for this graph is
\[
F(G, x) = \sum_{v \in V(G)} x^{deg(v)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
F(P_m + P_m, x) = 4x^4 + (8 + 4x)^2 + 8x^8 + 2x^2 + (2^2 + 6)x^8 + 40 - 1)x^3 + 2(2x + 2t - 1)x^3 + 2t - 1)x^2.
\]

**Proof.** By definition of forgotten polynomial, we have:
\[
F(G, x) = \sum_{v \in V(G)} x^{deg(v)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + T_m)} x^{deg(v) + deg(w)}
\]
\[
F(P_m + P_m, x) = 4x^4 + (8 + 4x)^2 + 8x^8 + 2x^2 + (2^2 + 6)x^8 + 40 - 1)x^3 + 2(2x + 2t - 1)x^3 + 2t - 1)x^2.
\]

The graph of first, second Zagreb and Forgotten polynomials for \( P_m + T_m \) are shown in Figure 7 Below.

**Proposition:** Let \( G \cong P_m + T_m \) be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are
\[
BM(P_m + T_m) = 656 + 49(8 + 4x)^2 + 81(2r + 64)(2^2 + 2 - 6) + 324(t - 1) + 200(t - 1) + 242 + 288x(t - 1);
\]
\[
PM_1(P_m + T_m) = 6^2 + 7(8 + 4x)^2 + 8x^2 + 8(2^2 + 2 - 6) + (9^2(t - 1) + 102x + 10x^2 + 12(t - 1); t - 1);
\]
\[
PM_2(P_m + T_m) = 9^2 + 15(8 + 4x)^2 + 158 + 18(t - 1) + 16(2^2 + 2 - 6);
\]
\[
F(P_m + T_m, x) = 18^2 + 25(8 + 4x)^2 + 34^2 + 45(2^2 + 2 - 6) + (4 + 1)^2(t - 1) = 52(2^2 + 2 - 6) + 50(t - 1) + 61(t - 1) + 72(t - 1).
\]

**Proof.** (1) Let \( G \cong P_m + P_m \) be the graph then by equation (1), we have:
\[
BM(P_m + P_m) = \sum_{v \in V(G)} x^{deg(v)} + \sum_{w \in V(G)} x^{deg(w)} + \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)}
\]
\[
+ \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)}
\]
\[
+ \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)} + \sum_{(v, w) \in E(P_m + P_m)} x^{deg(v) + deg(w)}
\]
\[
BM(P_m + P_m) = 656 + 49(8 + 4x)^2 + 81(2r + 64)(2^2 + 2 - 6) + 324(t - 1) + 200(t - 1) + 242 + 288x(t - 1);
\]

(2) By definition
Some topological descriptors algebraic polynomials, second Zagreb polynomial and Forgotten polynomial, and also closed forms of topological indices such as hyper Zagreb index, first Zagreb index, second Zagreb index and forgotten index for \( P_{m+r}P_m \) graphs.

**Conclusion**

In this article, we computed first Zagreb polynomial, second Zagreb polynomial and Forgotten polynomial, and also closed forms of topological indices such as hyper Zagreb polynomial. Some topological descriptors algebraic polynomials, Zagreb and Forgotten indices are shown in Figure 8.

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