

FULL PAPER

QSPR analysis on graph energies of chemical graphs

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Chemical graph theory is a bridge between Chemistry and Graph theory. Graph energies are important tools for QSPR researches. Thus, this study aims to relate several energies with the QSPR analysis of 67 alkanes. We compare these results with the Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy. Our study reveals some interesting results based on the predicting power of these graph energies.

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KEYWORDS

QSPR-analysis; octane isomers; Maximum Degree Energy; Minimum Degree Energy; Second Zagreb Energy.

Introduction

Graph theory is not only the study of different properties of objects, but also speaks about the objects with the same properties. Graph theory is used to investigate the properties through Quantitative Structure- Activity Relationship (QSAR) and Quantitative Structure- Property Relationship (QSPR) models through graph invariants such as topological indices. The concept of topological index while working on boiling point was firstly introduced by Wiener in 1947. The topological indices have found applications in different areas of mathematics, physics, biology, chemistry, informatics, etc. [6,10,23], but their most important use is in the non-empirical and Quantitative Structure -Activity Relationships (QSAR), and Quantitative Structure- Property Relationships (QSPR) [1,2,3,8,12,13,15,16,21,22,24].

Octane isomers

Octane isomers are the important set of organic molecules to test the applicability of various topological parameters in quantitative structure-property/activity relationships

(QSPR/QSAR). These compounds structurally differ enough to yield considerable variation in shape, branching, and non-polarity [19]. In a comprehensive study of numerous properties of octane isomers, Randic *et al.* [17,18,19] have used single molecular descriptors and concluded that different physicochemical properties depend on different descriptors.

In a molecular graph, chemical bond between them and carbon atoms in a hydrocarbon system represent edges and vertices, respectively. A plenty of work has been done on chemical graph theory and graph energies. Certain elementary results on the graph's energy are described in the thesis of Siraj [20].

Let G be a finite, simple, and undirected graph and its vertex set and edge set are given by $V(G) = \{v_1, v_2, v_3, \dots, v_p\}$ and $E(G) = \{e_1, e_2, e_3, \dots, e_q\}$, respectively. In 1978, Ivan Gutman innovated a novel graph spectral quantity which he named graph energy [9]. Let G be a simple graph of order n . Let $A(G)$ be its adjacency matrix. The eigenvalues of $A(G)$, denoted by $\lambda_1, \lambda_2, \dots, \lambda_n$ form the spectrum of G [7].

Definition: The sum of the absolute values of all eigenvalues of the graph G is called the energy of the graph G and is given as $E(G) = \sum_{i=1}^n |\lambda_i|$

The following graph energies are considered for the QSPR study.

Maximum Degree Energy

In [4], the maximum degree energy E_M of a simple connected graph G is the sum of the absolute values of eigen values of the maximum degree matrix $M(G)$ of a graph G .

Then, $M(G) = [M_{ij}]$ where,

$$M_{ij} = \begin{cases} \max(d_i, d_j), & \text{if } v_i, v_j \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

Where, d_i and d_j are the degrees of vertices v_i and v_j , respectively.

Minimum Degree Energy

In [5], the minimum degree energy E_m of a simple connected graph G is the sum of the absolute values of eigenvalues of the minimum degree matrix $m(G)$ of a graph G .

Then, $m(G) = [m_{ij}]$ where,

$$m_{ij} = \begin{cases} \min(d_i, d_j), & \text{if } v_i, v_j \in E(G) \\ 0, & \text{otherwise} \end{cases}$$

Where, d_i and d_j are the degrees of vertices v_i and v_j , respectively.

Second Zagreb Energy

In [14], the Second Zagreb energy ZE_2 of a simple connected graph G is the sum of the absolute values of eigenvalues of the second Zagreb matrix $Z^{(2)}(G)$ of G .

Then, $Z^{(2)}(G) = [Z_{ij}^{(2)}]$ where,

$$Z_{ij}^{(2)} = \begin{cases} d_i d_j, & \text{if } v_i, v_j \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

Where, d_i and d_j are the degrees of vertices v_i and v_j , respectively.

The use of selected graph energies in QSPR studies

We have used three energies. M_{ij} , m_{ij} , $Z_{ij}^{(2)}$ for eight representative physical properties molar volumes (mv) at 20°C, molar refractions (mr) at 20°C, boiling points (bp), surface tensions (st) at 20°C, melting points (mp), critical temperature (ct), critical Pressure (cp) and heats of vaporization (hv) at 25 °C of the 67 alkanes from n-butanenes to nonanes are listed in Table 1 and values for these properties were taken from [11].

TABLE 1

| Sl No. | Alkane | bp(°C) | mv(cm ³) | mr(cm ³) | hv(kJ) | ct(°C) | cp(atm) | st($\frac{\text{dyne}}{\text{cm}}$) | mp(°C) |
|--------|---------------------|--------|----------------------|----------------------|--------|--------|---------|---------------------------------------|---------|
| 1 | Butane | -0.5 | | | | 152.01 | 37.47 | | -138.35 |
| 2 | 2-methyl propane | -11.73 | | | | 134.98 | 36 | | -159.6 |
| 3 | Pentane | 36.074 | 115.205 | 25.2656 | 26.42 | 196.62 | 33.31 | 16 | -129.72 |
| 4 | 2-methyl butane | 27.852 | 116.426 | 25.2923 | 24.59 | 187.7 | 32.9 | 15 | -159.9 |
| 5 | 2,2-dimethylpropane | 9.503 | 112.074 | 25.7243 | 21.78 | 160.6 | 31.57 | | -16.55 |
| 6 | Hexane | 68.74 | 130.688 | 29.9066 | 31.55 | 234.7 | 29.92 | 18.42 | -95.35 |
| 7 | 2-methylpentane | 60.271 | 131.933 | 29.9459 | 29.86 | 224.9 | 29.95 | 17.38 | -153.67 |
| 8 | 3-methylpentane | 63.282 | 129.717 | 30 | 30.27 | 231.2 | 30.83 | 18.12 | -118 |
| 9 | 2,2-methylbutane | 49.741 | 132.744 | 29.9347 | 27.69 | 216.2 | 30.67 | 16.3 | -99.87 |
| 10 | 2,3-dimethylbutane | 57.988 | 130.24 | 29.8104 | 29.12 | 227.1 | 30.99 | 17.37 | -128.54 |
| 11 | Heptanes | 98.427 | 146.54 | 34.5504 | 36.55 | 267.55 | 27.01 | 20.26 | -90.61 |
| 12 | 2-methylhexane | 90.052 | 147.656 | 34.5908 | 34.8 | 257.9 | 27.2 | 19.29 | -118.28 |
| 13 | 3-methylhexane | 91.85 | 145.821 | 34.4597 | 35.08 | 262.4 | 28.1 | 19.79 | -119.4 |
| 14 | 3-ethylpentane | 93.475 | 143.517 | 34.2827 | 35.22 | 267.6 | 28.6 | 20.44 | -118.6 |
| 15 | 2,2-dimethylpentane | 79.197 | 148.695 | 34.6166 | 32.43 | 247.7 | 28.4 | 18.02 | -123.81 |

| | | | | | | | | | |
|----|------------------------|---------|---------|---------|-------|--------|-------|-------|---------|
| 16 | 2,3-dimethylpentane | 89.784 | 144.153 | 34.3237 | 34.24 | 264.6 | 29.2 | 19.96 | -119.24 |
| 17 | 2,4-dimethylpentane | 80.5 | 148.949 | 34.6192 | 32.88 | 247.1 | 27.4 | 18.15 | -119.24 |
| 18 | 3,3-dimethylpentane | 86.064 | 144.53 | 34.3323 | 33.02 | 263 | 30 | 19.59 | -134.46 |
| 19 | Octane | 125.665 | 162.592 | 39.1922 | 41.48 | 296.2 | 24.64 | 21.76 | -56.79 |
| 20 | 2-methylheptane | 117.647 | 163.663 | 39.2316 | 39.68 | 288 | 24.8 | 20.6 | -109.04 |
| 21 | 3-methylheptane | 118.925 | 161.832 | 39.1001 | 39.83 | 292 | 25.6 | 21.17 | -120.5 |
| 22 | 4-methylheptane | 117.709 | 162.105 | 39.1174 | 39.67 | 290 | 25.6 | 21 | -120.95 |
| 23 | 3-ethylhexane | 118.53 | 160.07 | 38.94 | 39.4 | 292 | 25.74 | 21.51 | |
| 24 | 2,2-dimethylhexane | 10.84 | 164.28 | 39.25 | 37.29 | 279 | 25.6 | 19.6 | -121.18 |
| 25 | 2,3-dimethylhexane | 115.607 | 160.39 | 38.98 | 38.79 | 293 | 26.6 | 20.99 | |
| 26 | 2,4-dimethylhexane | 109.42 | 163.09 | 39.13 | 37.76 | 282 | 25.8 | 20.05 | -137.5 |
| 27 | 2,5-dimethylhexane | 109.1 | 164.69 | 39.25 | 37.86 | 279 | 25 | 19.73 | -91.2 |
| 28 | 3,3-dimethylhexane | 111.96 | 160.87 | 39 | 37.93 | 290.84 | 27.2 | 20.63 | -126.1 |
| 29 | 3,4-dimethylhexane | 117.72 | 158.81 | 38.84 | 39.02 | 298 | 27.4 | 21.64 | |
| 30 | 3-ethyl-2-thylpentane | 115.65 | 158.79 | 38.83 | 38.52 | 295 | 27.4 | 21.52 | -114.96 |
| 31 | 3-ethyl-3-thylpentane | 118.25 | 157.02 | 38.71 | 37.99 | 305 | 28.9 | 21.99 | -90.87 |
| 32 | 2,2,3-trimethylpentane | 109.84 | 159.52 | 38.92 | 36.91 | 294 | 28.2 | 20.67 | -112.27 |
| 33 | 2,2,4-trimethylpentane | 99.23 | 165.08 | 39.26 | 35.13 | 271.15 | 25.5 | 18.77 | -107.38 |
| 34 | 2,3,3-trimethylpentane | 114.76 | 157.29 | 38.76 | 37.22 | 303 | 29 | 21.56 | -100.7 |
| 35 | 2,3,4-trimethylpentane | 113.46 | 158.85 | 38.56 | 37.61 | 295 | 27.6 | 21.14 | -109.21 |
| 36 | Nonane | 150.79 | 178.71 | 43.84 | 46.44 | 322 | 22.74 | 22.92 | -53.52 |
| 37 | 2-methyloctane | 143.26 | 179.77 | 43.87 | 44.65 | 315 | 23.6 | 21.88 | -80.4 |
| 38 | 3-methyloctane | 144.18 | 177.95 | 43.72 | 44.75 | 318 | 23.7 | 22.34 | -107.64 |
| 39 | 4-methyloctane | 142.48 | 178.15 | 43.76 | 44.75 | 318.3 | 23.06 | 22.34 | -113.2 |
| 40 | 3-ethylheptane | 143 | 176.41 | 43.64 | 44.81 | 318 | 23.98 | 22.81 | -114.9 |
| 41 | 4-ethylheptane | 141.20 | 175.68 | 43.49 | 44.81 | 318.3 | 23.98 | 22.81 | |
| 42 | 2,2-dimethylheptane | 132.69 | 180.5 | 43.91 | 42.28 | 302 | 22.8 | 20.8 | -113 |
| 43 | 2,3-dimethylheptane | 140.5 | 176.65 | 43.63 | 43.79 | 315 | 23.79 | 22.34 | -116 |
| 44 | 2,4-dimethylheptane | 133.5 | 179.12 | 43.73 | 42.87 | 306 | 22.7 | 23.3 | |
| 45 | 2,5-dimethylheptane | 136 | 179.37 | 43.84 | 43.87 | 307.8 | 22.7 | 21.3 | |
| 46 | 2,6-dimethylheptane | 135.21 | 180.91 | 43.92 | 42.82 | 306 | 23.7 | 20.83 | -102.9 |
| 47 | 3,3-dimethylheptane | 137.3 | 176.897 | 43.687 | 42.66 | 314 | 24.19 | 22.01 | |
| 48 | 3,4-dimethylheptane | 140.6 | 175.349 | 43.5473 | 43.84 | 322.7 | 24.77 | 22.8 | |
| 49 | 3,5-dimethylheptane | 136 | 177.386 | 43.6379 | 42.98 | 312.3 | 23.59 | 21.77 | |
| 50 | 4,4-dimethylheptane | 135.2 | 176.897 | 43.6022 | 42.66 | 317.8 | 24.18 | 22.01 | |
| 51 | 3-ethyl-2-ethylhexane | 138 | 175.445 | 43.655 | 43.84 | 322.7 | 24.77 | 22.8 | |
| 52 | 4-ethyl-2-ethylhexane | 133.8 | 177.386 | 43.6472 | 42.98 | 330.3 | 25.56 | 21.77 | |
| 53 | 3-ethyl-3-ethylhexane | 140.6 | 173.077 | 43.268 | 44.04 | 327.2 | 25.66 | 23.22 | |

| | | | | | | | | | |
|----|-----------------------------|---------|---------|---------|-------|-------|-------|-------|---------|
| 54 | 2,2,4-trimethylhexane | 126.54 | 179.22 | 43.7638 | 40.57 | 301 | 23.39 | 20.51 | -120 |
| 55 | 2,2,5-trimethylhexane | 124.084 | 181.346 | 43.9356 | 40.17 | 296.6 | 22.41 | 20.04 | -105.78 |
| 56 | 2,3,3-trimethylhexane | 137.68 | 173.78 | 43.4347 | 42.23 | 326.1 | 25.56 | 22.41 | -116.8 |
| 57 | 2,3,4-trimethylhexane | 139 | 173.498 | 43.4917 | 42.93 | 324.2 | 25.46 | 22.8 | |
| 58 | 2,3,5-trimethylhexane | 131.34 | 177.656 | 43.6474 | 41.42 | 309.4 | 23.49 | 21.27 | -127.8 |
| 59 | 3,3,4-trimethylhexane | 140.46 | 172.055 | 43.3407 | 42.28 | 330.6 | 26.45 | 23.27 | -101.2 |
| 60 | 3,3-diethylpentane | 146.168 | 170.185 | 43.1134 | 43.36 | 342.8 | 26.94 | 23.75 | -33.11 |
| 61 | 2,2-dimethyl-3-ethylpentane | 133.83 | 174.537 | 43.4571 | 42.02 | 322.6 | 25.96 | 22.38 | -99.2 |
| 62 | 2,3-dimethyl-3-ethylpentane | 142 | 170.093 | 42.9542 | 42.55 | 338.6 | 26.94 | 23.87 | |
| 63 | 2,4-dimethyl-3-ethylpentane | 136.73 | 173.804 | 43.4037 | 42.93 | 324.2 | 25.46 | 22.8 | -122.2 |
| 64 | 2,2,3,3-tetramethylpentane | 140.274 | 169.495 | 43.2147 | 41 | 334.5 | 27.04 | 23.38 | -99 |
| 65 | 2,2,3,4-tetramethylpentane | 133.016 | 173.557 | 43.4359 | 41 | 319.6 | 25.66 | 21.98 | -121.09 |
| 66 | 2,2,4,4-tetramethylpentane | 122.284 | 178.256 | 43.8747 | 38.1 | 301.6 | 24.58 | 20.37 | -66.54 |
| 67 | 2,3,3,4-tetramethylpentane | 141.551 | 169.928 | 43.2016 | 41.75 | 334.5 | 26.85 | 23.31 | -102.12 |

TABLE 2

| | Alkanes | M_{ij} | m_{ij} | Z_{ij}^2 |
|----|---------------------|----------|----------|------------|
| 1 | Butane | 0.894 | 5.656 | 11.313 |
| 2 | 2-methyl propane | 10.392 | 3.461 | 10.392 |
| 3 | Pentane | 10.928 | 8 | 16 |
| 4 | 2-methyl butane | 15.7006 | 6.269 | 17.316 |
| 5 | 2,2 dimethylpropane | 16 | 4 | 16 |
| 6 | Hexane | 13.976 | 10.724 | 22.976 |
| 7 | 2-methylpentane | 15.726 | 8.598 | 21.808 |
| 8 | 3-methylpentane | 17.114 | 7.656 | 23.697 |
| 9 | 2,2-methylbutane | 19.566 | 6.771 | 23.975 |
| 10 | 2,3-dimethylbutane | 18 | 8.246 | 24.738 |
| 11 | Heptanes | 16.109 | 14.393 | 26.221 |
| 12 | 2-methylhexane | 16.58 | 10.724 | 27.362 |
| 13 | 3-methylhexane | 19.098 | 11.328 | 28.386 |
| 14 | 3-ethylpentane | 18.582 | 11.211 | 29.166 |
| 15 | 2,2-dimethylpentane | 21.385 | 9.103 | 28.407 |
| 16 | 2,3-dimethylpentane | 21.214 | 10.81 | 26.832 |
| 17 | 2,4-dimethylpentane | 20.485 | 9.152 | 27.458 |
| 18 | 3,3-dimethylpentane | 23.039 | 9.437 | 31.322 |
| 19 | Octane | 19.036 | 15.81 | 30.539 |
| 20 | 2-methylheptane | 19.566 | 13.708 | 32.056 |
| 21 | 3-methylheptane | 18.624 | 11.206 | 32.852 |
| 22 | 4-methylheptane | 21.018 | 13.931 | 32.681 |
| 23 | 3-ethylhexane | 22.128 | 14.606 | 37.789 |
| 24 | 2,2-dimethylhexane | 24.564 | 11.864 | 33.987 |
| 25 | 2,3-dimethylhexane | 23.22 | 13.24 | 35.594 |
| 26 | 2,4-dimethylhexane | 21.83 | 11.328 | 32.471 |
| 27 | 2,5-dimethylhexane | 23.466 | 11.232 | 30.802 |
| 28 | 3,3-dimethylhexane | 24.906 | 11.814 | 33.25 |
| 29 | 3,4-dimethylhexane | 20.39 | 11.972 | 33.374 |

| | | | | |
|----|----------------------------|---------|---------|--------|
| 30 | 3-ethyl-2-methylpentane | 23.033 | 13.025 | 34.395 |
| 31 | 3-ethyl-3-methylpentane | 26.33 | 11.999 | 34.862 |
| 32 | 2,2,3-trimethylpentane | 25.518 | 9.12 | 21.349 |
| 33 | 2,2,4-trimethylpentane | 26.122 | 9.632 | 33.963 |
| 34 | 2,3,3-trimethylpentane | 27.176 | 19.653 | 39.74 |
| 35 | 2,3,4-trimethylpentane | 25.276 | 12.662 | 40.521 |
| 36 | Nonane | 19.034 | 15.269 | 36.427 |
| 37 | 2-methyloctane | 23.913 | 15.788 | 37.525 |
| 38 | 3-methyloctane | 24.28 | 15.678 | 38.599 |
| 39 | 4-methyloctane | 24.121 | 15.179 | 38.483 |
| 40 | 3-ethylheptane | 24.302 | 15.9708 | 37.557 |
| 41 | 4-ethylheptane | 24.255 | 16.209 | 39.281 |
| 42 | 2,2-dimethylheptane | 26.611 | 14.21 | 38.659 |
| 43 | 2,3-dimethylheptane | 26.256 | 15.932 | 41.015 |
| 44 | 2,4-dimethylheptane | 25.788 | 14.235 | 38.658 |
| 45 | 2,5-dimethylheptane | 27.038 | 14.681 | 39.76 |
| 46 | 2,6-dimethylheptane | 25.754 | 13.1104 | 36.196 |
| 47 | 3,3-dimethylheptane | 28.047 | 14.547 | 55.819 |
| 48 | 3,4-dimethylheptane | 26.435 | 15.266 | 41.836 |
| 49 | 3,5-dimethylheptane | 20.428 | 14.6008 | 40.68 |
| 50 | 4,4-dimethylheptane | 17.906 | 14.166 | 40.432 |
| 51 | 3-ethyl-2-methylhexane | 26.168 | 15.283 | 41.51 |
| 52 | 4-ethyl-2-methylhexane | 24.842 | 13.412 | 37.681 |
| 53 | 3-ethyl-3-methylhexane | 28.297 | 13.723 | 42.937 |
| 54 | 2,2,4- trimethyl hexane | 29.535 | 11.377 | 40.637 |
| 55 | 2,2,5- trimethyl hexane | 29.465 | 12.523 | 39.963 |
| 56 | 2,3,3- trimethyl hexane | 29.052 | 12.648 | 44.383 |
| 57 | 2,3,4- trimethyl hexane | 28.5006 | 15.231 | 44.165 |
| 58 | 2,3,5- trimethyl hexane | 28.005 | 12.802 | 39.717 |
| 59 | 3,3,4- trimethyl hexane | 30.387 | 13.347 | 45.948 |
| 60 | 3,3-diethylpentane | 24.92 | 12.246 | 44.249 |
| 61 | 2,2-dimethyl-3-thylpentane | 28.733 | 13.488 | 39.798 |
| 62 | 2,3-dimethyl-3-thylpentane | 28.5106 | 13.655 | 42.126 |
| 63 | 2,4-dimethyl-3-thylpentane | 26.813 | 17.376 | 42.776 |
| 64 | 2,2,3,3-tetramethylpentane | 32.278 | 12.408 | 49.07 |
| 65 | 2,2,3,4-tetramethylpentane | 30.993 | 12.484 | 45.548 |
| 66 | 2,2,4,4-tetramethylpentane | 31.744 | 10.097 | 40.389 |
| 67 | 2,3,3,4-tetramethylpentane | 31.288 | 12.305 | 47.778 |

Regression Models: The following statistical models have been used for the study:

1) Linear Model:

$$P = A + B(TI)$$

2) Quadratic model:

$$P = A + B(TI) + C(TI)^2$$

Where, P is a physical property, TI is a topological index and A, B, and C are constants. The followings are different regression models for each topological index, which are listed below.

Linear Model:

1) Maximum degree energy $M_{ij}(G)$:

$$bp = -6.7005 + [M_{ij}(G)] 5.0387$$

$$mv = 97.1519 + [M_{ij}(G)] 2.7632$$

$$mr = 19.1559 + [M_{ij}(G)] 0.8593$$

$$hv = 22.0944 + [M_{ij}(G)] 0.7079$$

$$ct = 137.4623 + [M_{ij}(G)] 6.4755$$

$$cp = 35.2045 - [M_{ij}(G)] 0.3719$$

$$St = 14.6962 + [M_{ij}(G)] 0.2643$$

$$mp = -130.4421 + [M_{ij}(G)] 0.9453$$

2) Minimum degree energy $m_{ij}(G)$:

$$bp = -18.0655 + [m_{ij}(G)] 10.3757$$

$$mv = 103.6366 + [m_{ij}(G)] 4.6826$$

$$mr = 21.7496 + [m_{ij}(G)] 1.4101$$

$$hv = 18.4233 + [m_{ij}(G)] 1.6262$$

$$\begin{aligned}
 ct &= 136.386 + [m_{ij}(G)] 12.2311 \\
 cp &= 36.0431 - [m_{ij}(G)] 0.7657 \\
 St &= 13.5881 + [m_{ij}(G)] 0.5825 \\
 mp &= -131.2434 + [m_{ij}(G)] 1.8894
 \end{aligned}$$

3) Second Zagreb energy $Z_{ij}^{(2)}(G)$:

$$bp = -18.6006 + [Z_{ij}^{(2)}(G)] 3.7007$$

$$\begin{aligned}
 mv &= 97.2123 + [Z_{ij}^{(2)}(G)] 1.8425 \\
 mr &= 19.2455 + [Z_{ij}^{(2)}(G)] 0.571 \\
 hv &= 19.3908 + [Z_{ij}^{(2)}(G)] 0.5491 \\
 ct &= 128.3849 + [Z_{ij}^{(2)}(G)] 4.5759 \\
 cp &= 35.7149 - [Z_{ij}^{(2)}(G)] 0.2625 \\
 St &= 13.6247 + [Z_{ij}^{(2)}(G)] 0.2059 \\
 mp &= -134.2955 + [Z_{ij}^{(2)}(G)] 0.767
 \end{aligned}$$

Note: In tables 3 to 8, N is the total number of chemical structures used for estimation during analysis, r is the correlation coefficient, s is the standard error of estimate, and F (Fisher statistic) is the statistical test to know the goodness of fit for regression analysis.

TABLE 3 Statistical parameters for the linear QSPR model for the Maximum degree energy $M_{ij}(G)$

| Physical Properties | N | A | B | r | s | F |
|------------------------|----|-----------|---------|---------|---------|---------|
| Boiling point | 67 | -6.7005 | 5.0387 | 0.7348 | 26.9921 | 74.907 |
| Molar volume | 65 | 97.1519 | 2.7632 | 0.7483 | 11.9547 | 79.268 |
| Molar refraction | 65 | 19.1559 | 0.8593 | 0.78102 | 3.3316 | 98.710 |
| Heats of vapourization | 65 | 22.0944 | 0.7079 | 0.6244 | 4.2651 | 40.868 |
| Critical temperature | 67 | 137.4623 | 6.4755 | 0.8062 | 27.0638 | 123.064 |
| Critical pressure | 67 | 35.2045 | -0.3719 | 0.6782 | 2.3242 | 55.029 |
| Surface tension | 64 | 14.6962 | 0.2643 | 0.6403 | 1.5236 | 42.930 |
| Melting point | 52 | -130.4421 | 0.9453 | 0.2144 | 26.6437 | 2.429 |

TABLE 4 Statistical parameters for the linear QSPR model for the Minimum degree energy $m_{ij}(G)$

| Physical Properties | N | A | B | r | S | F |
|------------------------|----|-----------|---------|--------|---------|---------|
| Boiling point | 67 | -18.0655 | 10.3757 | 0.8246 | 22.3494 | 139.070 |
| Molar volume | 65 | 103.6366 | 4.6826 | 0.7483 | 11.8987 | 80.612 |
| Molar refraction | 65 | 21.7496 | 1.4101 | 0.7615 | 3.4732 | 85.789 |
| Heats of vapourization | 65 | 18.4233 | 1.6262 | 0.8544 | 2.8538 | 168.998 |
| Critical temperature | 67 | 136.386 | 12.2311 | 0.8366 | 25.1815 | 152.229 |
| Critical pressure | 67 | 36.0431 | -0.7657 | 0.7615 | 2.0382 | 91.073 |
| Surface tension | 64 | 13.5881 | 0.5825 | 0.7874 | 1.2171 | 102.433 |
| Melting point | 52 | -131.2434 | 1.8894 | 0.2302 | 26.556 | 2.776 |

TABLE 5 Statistical parameters for the linear QSPR model for the Second Zagreb energy $Z_{ij}^{(2)}(G)$

| Physical Properties | N | A | B | r | s | F |
|------------------------|----|-----------|---------|---------|---------|---------|
| Boiling point | 67 | -18.6006 | 3.7007 | 0.8544 | 65.4314 | 147.362 |
| Molar volume | 65 | 97.2123 | 1.8425 | 0.8485 | 9.5194 | 161.372 |
| Molar refraction | 65 | 19.2455 | 0.571 | 0.8831 | 2.4897 | 226.554 |
| Heats of vapourization | 65 | 19.3908 | 0.5491 | 0.8306 | 3.0618 | 138.560 |
| Critical temperature | 67 | 128.3849 | 4.5759 | 0.91104 | 18.9544 | 318.408 |
| Critical pressure | 67 | 35.7149 | -0.2625 | 0.7615 | 2.0457 | 89.928 |
| Surface tension | 64 | 13.6247 | 0.2059 | 0.8246 | 1.1133 | 134.520 |
| Melting point | 52 | -134.2955 | 0.767 | 0.2626 | 26.3276 | 3.696 |

Quadratic Model:

1) Maximum degree energy $M_{ij}(G)$:

$$bp = -39.7869 + 8.7194 [M_{ij}(G)] - 0.0919$$

$$[M_{ij}(G)]^2$$

$$mv = 8.6035 + 10.92508 [M_{ij}(G)] - 0.1793$$

$$[M_{ij}(G)]^2$$

$$\begin{aligned}
 mr &= -3.8685 + 2.9815 [M_{ij}(G)] - 0.0466 [M_{ij}(G)]^2 \\
 hv &= -6.4955 + 3.3431 [M_{ij}(G)] - 0.0579 [M_{ij}(G)]^2 \\
 ct &= 103.64006 + 10.23808 [M_{ij}(G)] - 0.0939 [M_{ij}(G)]^2 \\
 cp &= 41.0006 + 1.0166 [M_{ij}(G)] - 0.0161 [M_{ij}(G)]^2 \\
 St &= 7.4714 + 0.9278 [M_{ij}(G)] - 0.0145 [M_{ij}(G)]^2 \\
 mp &= -145.7607 + 2.6914 [M_{ij}(G)] - 0.0441 [M_{ij}(G)]^2
 \end{aligned}$$

2) Minimum degree energy $m_{ij}(G)$:

$$\begin{aligned}
 bp &= -116.3074 + 29.4347 [m_{ij}(G)] - 0.8466 [m_{ij}(G)]^2 \\
 mv &= 37.7159 + 16.5487 [m_{ij}(G)] - 0.5016 [m_{ij}(G)]^2 \\
 mr &= 3.1433 + 4.7593 [m_{ij}(G)] - 0.1416 [m_{ij}(G)]^2 \\
 hv &= -0.6825 + 5.0653 [m_{ij}(G)] - 0.1454 [m_{ij}(G)]^2 \\
 ct &= 1.5489 + 38.3896 [m_{ij}(G)] - 0.16208 [m_{ij}(G)]^2 \\
 cp &= 44.3739 - 2.3819 [m_{ij}(G)] + 0.0717 [m_{ij}(G)]^2 \\
 St &= 2.93306 + 2.3942 [m_{ij}(G)] - 0.0734 [m_{ij}(G)]^2
 \end{aligned}$$

$$mp = -116.595008 - 0.9876 [m_{ij}(G)] + 0.1289 [m_{ij}(G)]^2$$

3) Second Zagreb energy $Z_{ij}^{(2)}(G)$:

$$\begin{aligned}
 bp &= -100.9841 + 9.4004 [Z_{ij}^{(2)}(G)] - 0.0898 [Z_{ij}^{(2)}(G)]^2 \\
 mv &= 28.2939 + 6.1975 [Z_{ij}^{(2)}(G)] - 0.06463 [Z_{ij}^{(2)}(G)]^2 \\
 mr &= 1.5094 + 1.6917 [Z_{ij}^{(2)}(G)] - 0.0166 [Z_{ij}^{(2)}(G)]^2 \\
 hv &= -1.1473 + 1.8469 [Z_{ij}^{(2)}(G)] - 0.0192 [Z_{ij}^{(2)}(G)]^2 \\
 ct &= 28.3231 + 11.4987 [Z_{ij}^{(2)}(G)] - 0.1091 [Z_{ij}^{(2)}(G)]^2 \\
 cp &= 45.82906 - 0.9622 [Z_{ij}^{(2)}(G)] + 0.01102 [Z_{ij}^{(2)}(G)]^2 \\
 St &= 7.7847 + 0.5657 [Z_{ij}^{(2)}(G)] - 0.0052 [Z_{ij}^{(2)}(G)]^2 \\
 mp &= -145.02 + 1.5465 [Z_{ij}^{(2)}(G)] - 0.0128 [Z_{ij}^{(2)}(G)]^2
 \end{aligned}$$

TABLE 6 Statistical parameters for the quadratic QSPR model for the Maximum degree energy $M_{ij}(G)$

| Physical Properties | N | A | B | C | r | s | F |
|-----------------------|----|-----------|----------|---------|--------|--------|--------|
| Boiling point | 67 | -39.7869 | 8.7194 | -0.0919 | 0.7429 | 26.713 | 39.423 |
| Molar volume | 65 | 8.6035 | 10.92508 | -0.1793 | 0.7936 | 11.018 | 52.745 |
| Molar refraction | 65 | -3.8685 | 2.9815 | -0.0466 | 0.8161 | 3.109 | 61.830 |
| Heats of vaporization | 65 | -6.4955 | 3.3431 | -0.0579 | 0.6891 | 4 | 28.043 |
| Critical temperature | 67 | 103.64006 | 10.23808 | -0.0939 | 0.8168 | 26.764 | 64.148 |
| Critical pressure | 67 | 41.0006 | -1.0166 | 0.0161 | 0.7337 | 2.163 | 37.319 |
| Surface tension | 64 | 7.4714 | 0.9278 | -0.0145 | 0.6697 | 1.484 | 24.810 |
| Melting point | 52 | -145.7607 | 2.6914 | -0.0441 | 0.237 | 26.775 | 1.458 |

TABLE 7 Statistical parameters for the quadratic QSPR model for the Minimum degree energy $m_{ij}(G)$

| Physical Properties | N | A | B | C | r | s | F |
|-----------------------|----|----------|---------|---------|--------|--------|---------|
| Boiling point | 67 | -116.307 | 29.4347 | -0.8466 | 0.8789 | 19.037 | 108.639 |
| Molar volume | 65 | 37.7159 | 16.5487 | -0.5016 | 0.822 | 10.312 | 64.598 |
| Molar refraction | 65 | 3.1433 | 4.7593 | -0.1416 | 0.8245 | 3.045 | 65.822 |
| Heats of vaporization | 65 | -0.6825 | 5.0653 | -0.1454 | 0.912 | 2.264 | 153.367 |
| Critical temperature | 67 | 1.5489 | 38.3896 | -1.162 | 0.9097 | 19.262 | 153.624 |
| Critical pressure | 67 | 44.3739 | -2.3819 | 0.0717 | 0.8284 | 1.783 | 70.035 |
| Surface tension | 64 | 2.933 | 2.3942 | -0.0734 | 0.8686 | 0.99 | 93.796 |
| Melting point | 52 | -116.595 | -0.9876 | 0.1289 | 0.2421 | 26.751 | 1.505 |

TABLE 8 Statistical parameters for the quadratic QSPR model for the Second Zagreb energy $Z_{ij}^{(2)}(G)$

| Physical Properties | N | A | B | C | r | s | F |
|------------------------|----|-----------|---------|----------|---------|--------|---------|
| Boiling point | 67 | -100.9841 | 9.4004 | -0.0898 | 0.8945 | 17.841 | 128.114 |
| Molar volume | 65 | 28.2939 | 6.1975 | -0.0646 | 0.9089 | 7.55 | 147.354 |
| Molar refraction | 65 | 1.5094 | 1.6917 | -0.0166 | 0.9288 | 1.994 | 194.742 |
| Heats of vapourization | 65 | -1.1473 | 1.8469 | -0.01926 | 0.8886 | 2.532 | 116.382 |
| Critical temperature | 67 | 28.3231 | 11.4987 | -0.1091 | 0.95008 | 14.474 | 296.762 |
| Critical pressure | 67 | 45.829 | -0.9622 | 0.01102 | 0.8585 | 1.632 | 89.745 |
| Surface tension | 64 | 7.7847 | 0.5657 | -0.0052 | 0.8578 | 1.027 | 84.995 |
| Melting point | 52 | -145.02 | 1.5465 | -0.0128 | 0.2668 | 26.561 | 1.878 |

Discussion and concluding remarks

We have tested the model described in Table 1 for 67 alkanes from n-butanes to nonanes for linear and quadratic regression models of graph energies.

By inspection of the data given in Tables 3 to 8 as a result of QSPR analysis, it is desirable to extract the number of conclusions for the graph energies.

First, the Maximum Degree Energy is correlated, to predict physical properties of alkanes. In linear regression model, we can see that the correlation coefficient value ranges from 0.2144 to 0.8062. It is good in Critical temperature(ct) of alkanes with correlation coefficient $r=0.806$. It is poor in other physical properties.

In linear model of the Minimum Degree Energy, we can see that correlation coefficient value ranges from 0.2302 to 0.8544. It is good especially in Boiling point(Bp), Heat of vaporization(hv), and Critical temperature(ct) of alkanes with correlation coefficient $r=0.8246$, 0.8544 , and 0.8366 , respectively. The correlation is poor for other physical properties.

In linear regression model of the Second Zagreb Energy, we can see that the correlation coefficient value ranges from 0.2626 to 0.911. It is high especially in Critical temperature(ct) with $r=0.911$. We can use the Second Zagreb Energy as a tool for predicting physical properties of alkanes.

In quadratic model, we can see that correlation coefficient value varies from 0.237 to 0.8168. It is good especially in Molar refraction(mr) and Critical temperature(ct) of alkanes with correlation coefficient $r=0.8161$ and 0.8168 respectively. The correlation is poor for other physical properties.

In quadratic model, we can see that correlation coefficient value varies from 0.2421 to 0.912. However, it is very high in Heat of vaporization (hv) and Critical temperature(ct) with $r=0.912$ and 0.9097 , we can use Minimum Degree Energy as a tool to predict the physical properties of alkanes.

In quadratic model, we can see that the correlation coefficient value varies from 0.2668 to 0.9288. It is high especially in Molar volume (mv), Molar refraction(mr), and Boiling point(Bp) of alkanes. The correlation is

poor for melting points of alkanes, with correlation coefficient value of $r = 0.2668$.

However, in comparison with linear and quadratic models, the correlation is high in quadratic model for all physical properties and weak correlation to Melting point (mp) in linear and quadratic models.

If the absolute values of correlation coefficient are less than 0.8 can be characterized as useless, from the practical point of view graph energies.

Future scope

The QSPR study has revealed that the molecular descriptors are best to predict the

physicochemical properties of alkanes from n-butanes to nonane. The chemical people can use these topological indices for experimental data verification or they can reduce time by using these results directly in their experimentation. These graph energies will lead to many new applications in the chemical fields.

Correlation of graph energies with physico-chemical properties of 67 alkanes

In the following Figures, the correlation of Graph energies with physical properties of alkanes are demonstrated.

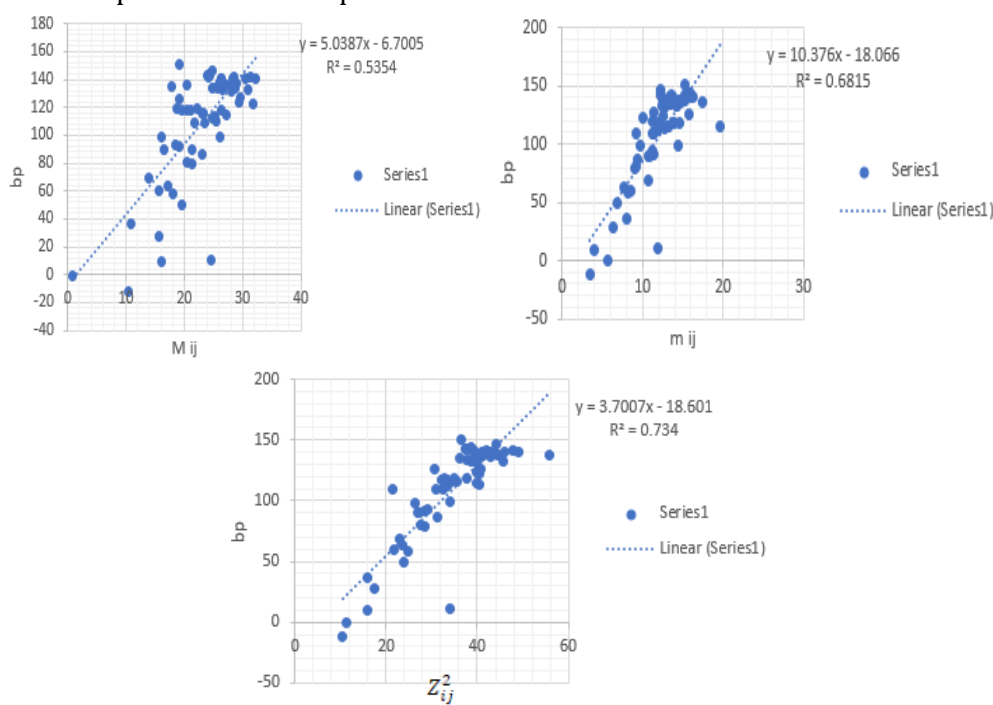


FIGURE 1 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (bp) with physical properties of 67 alkanes in linear model

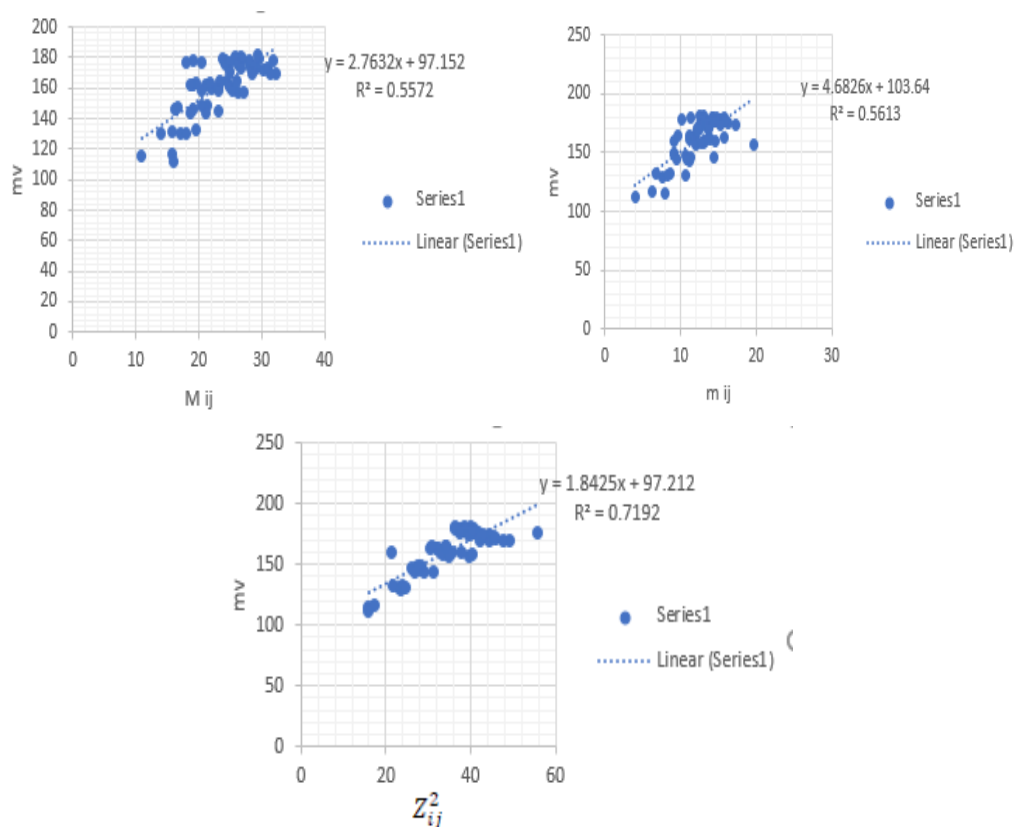


FIGURE 2 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (mv) with physical properties of 67 alkanes in linear model

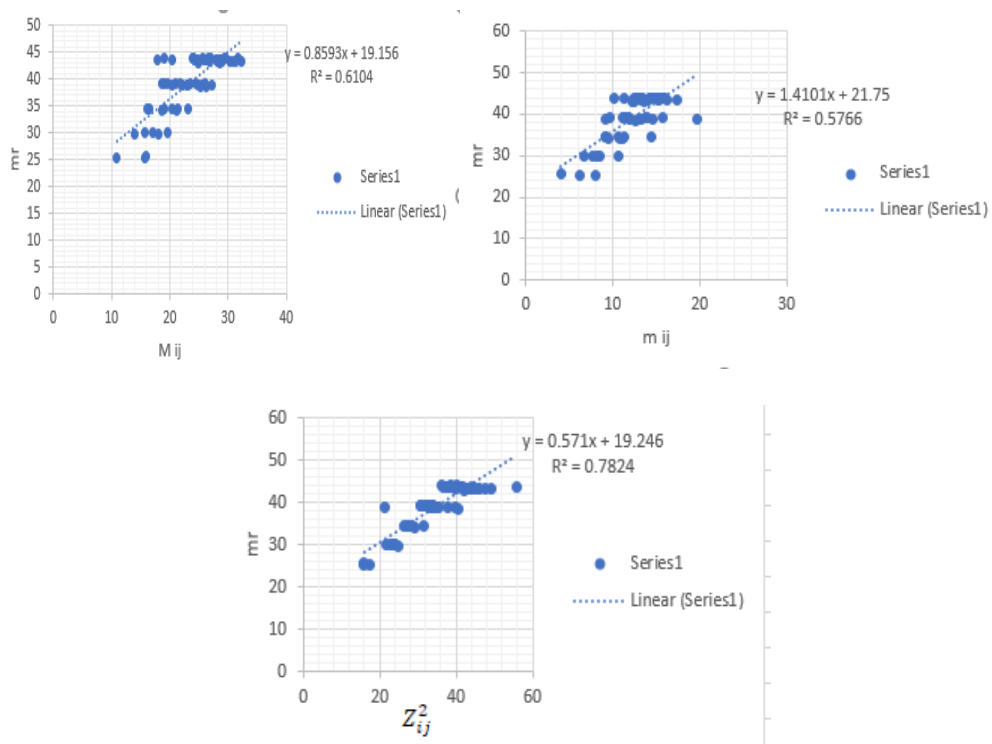


FIGURE 3 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (mr) with physical properties of 67 alkanes in linear model

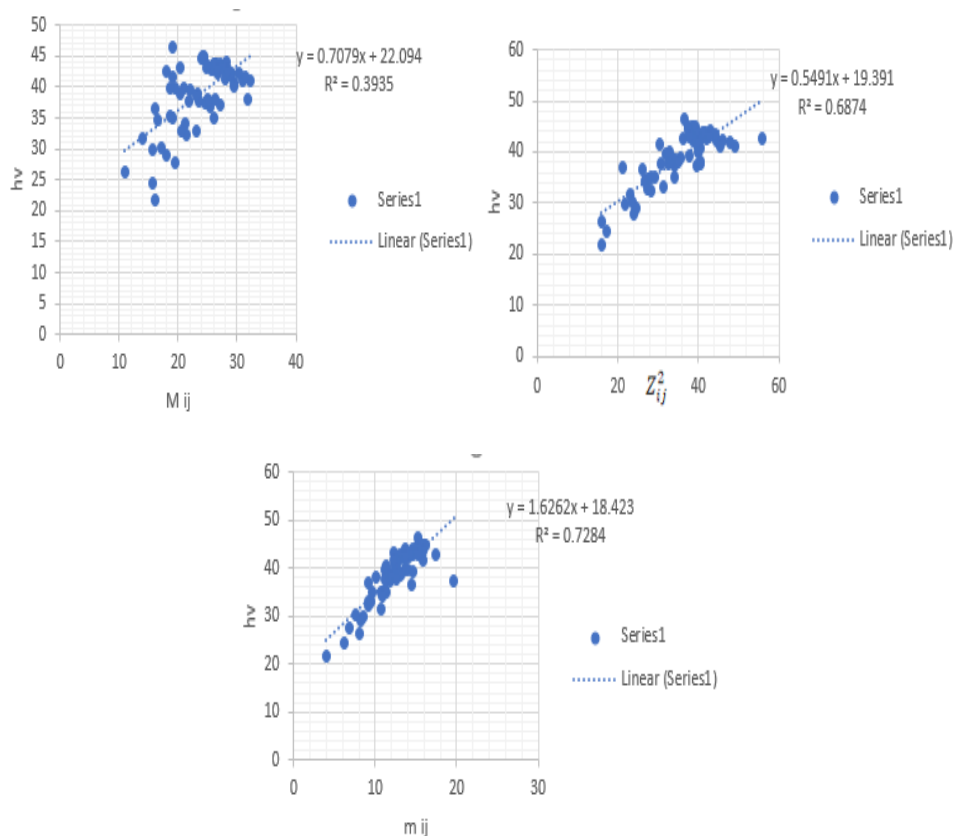


FIGURE 4 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy ($h\nu$) with physical properties of 67 alkanes in linear model

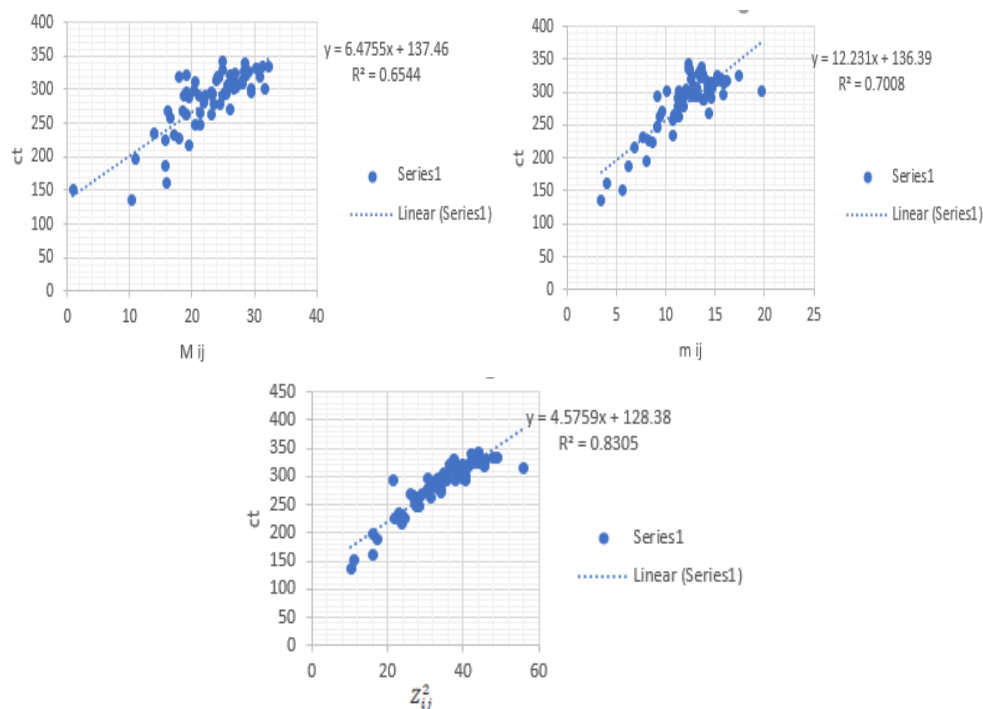


FIGURE 5 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (ct) with physical properties of 67 alkanes in linear model

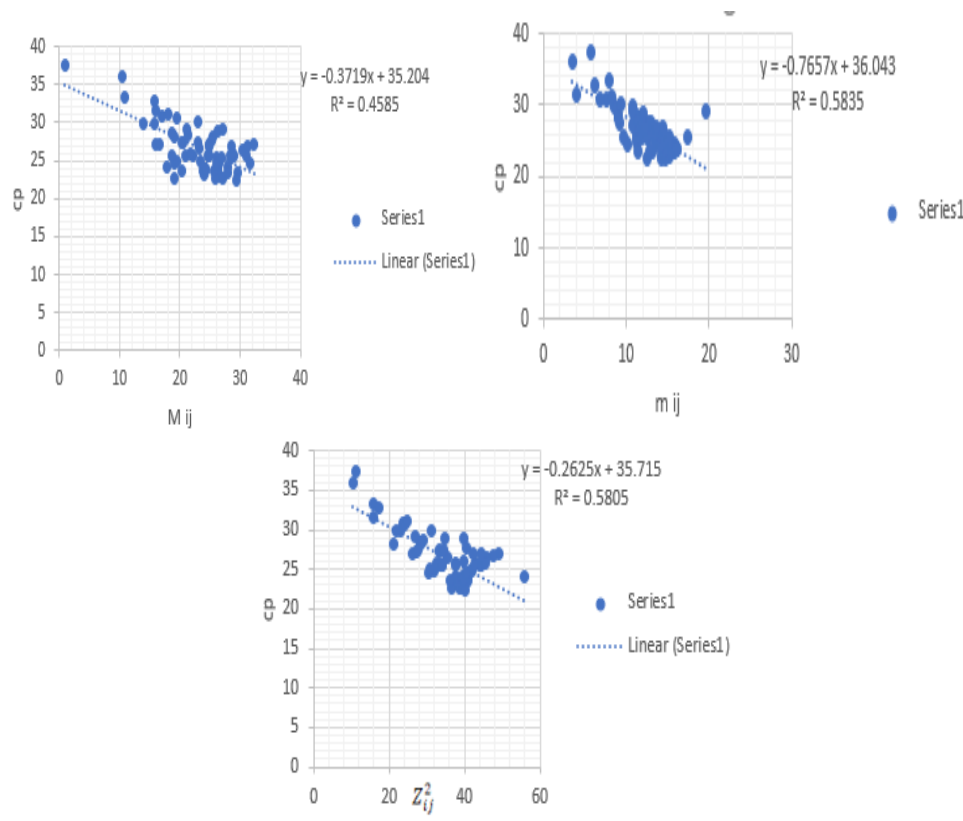


FIGURE 6 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (cp) with physical properties of 67 alkanes in linear model

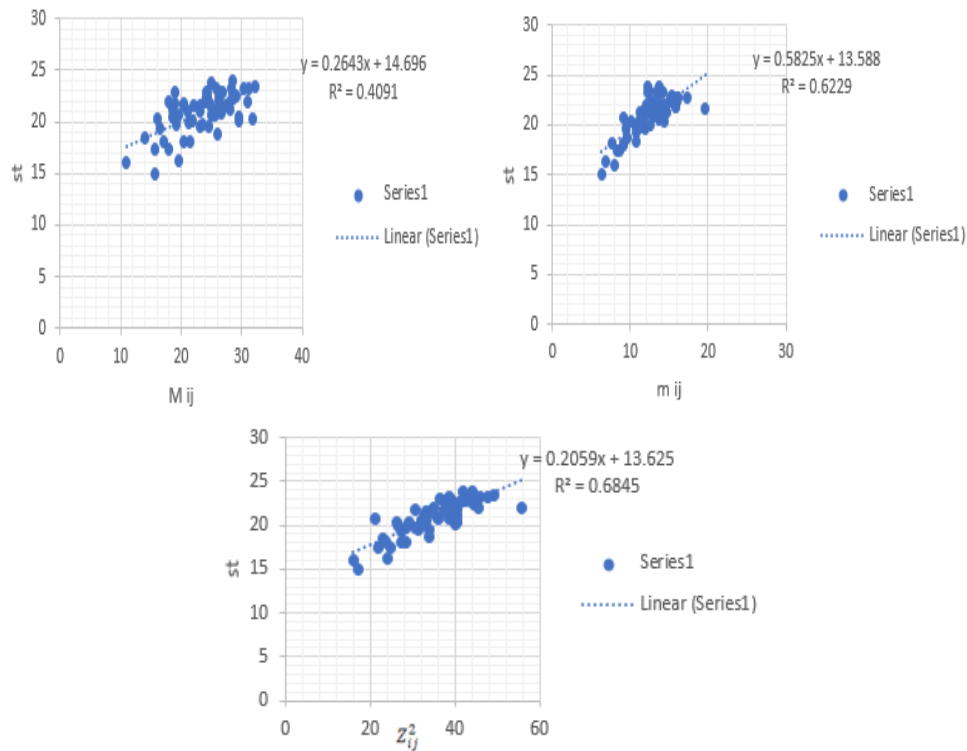


FIGURE 7 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (st) with physical properties of 67 alkanes in linear model

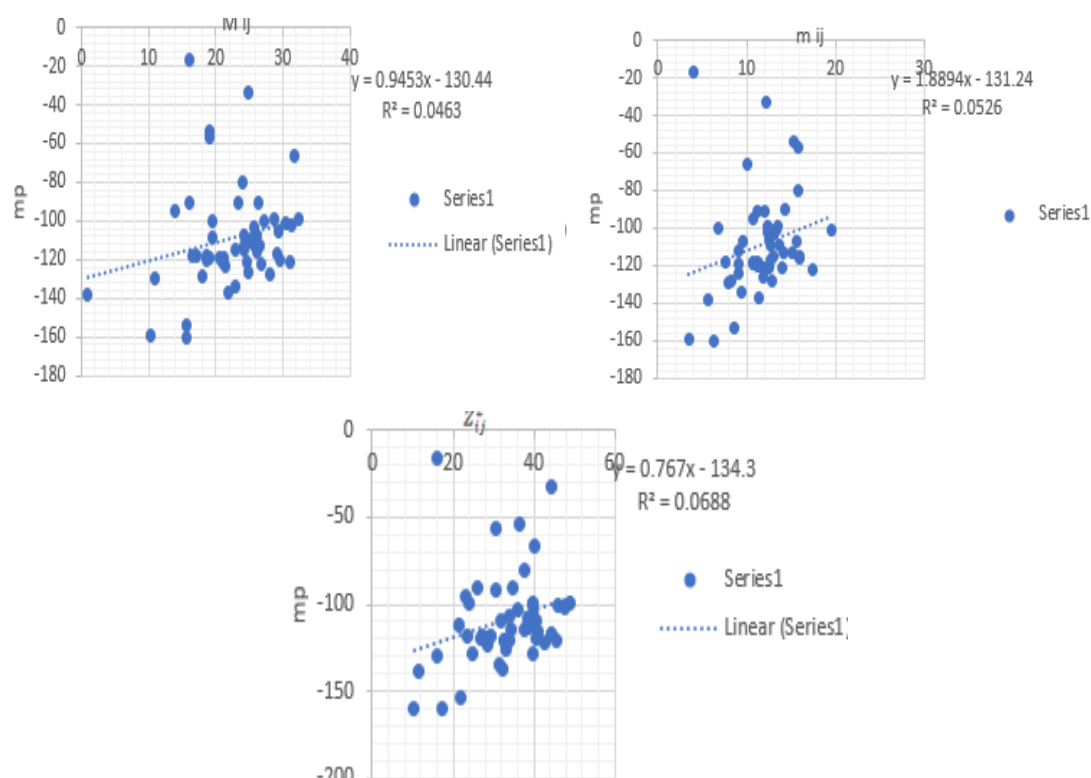


FIGURE 8 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (mp) with physical properties of 67 alkanes in linear model

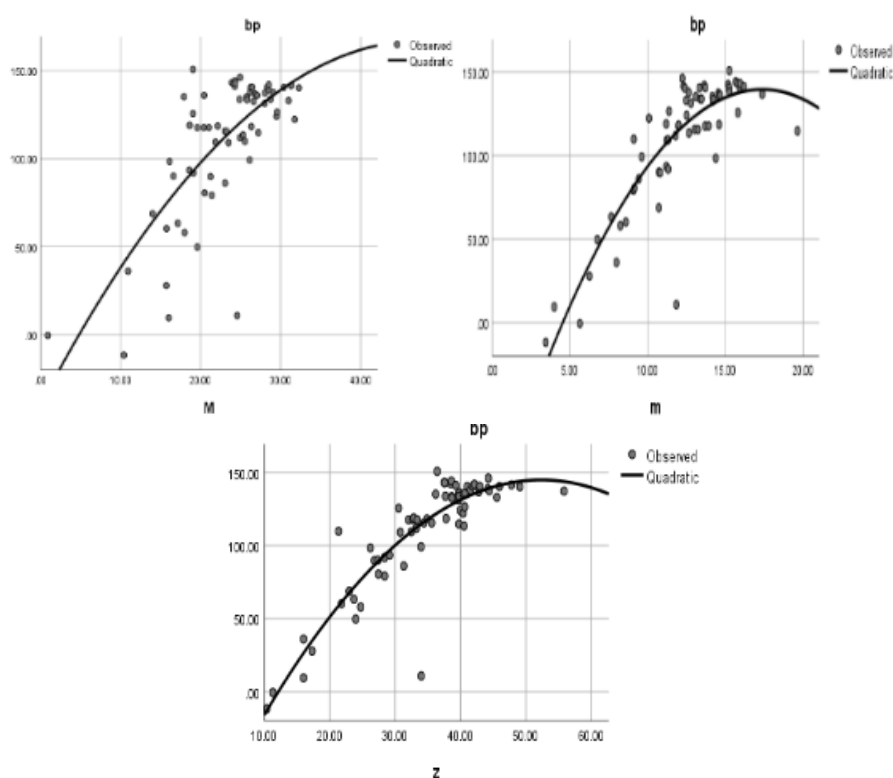


FIGURE 9 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (bp) with physical properties of 67 alkanes in Quadratic model

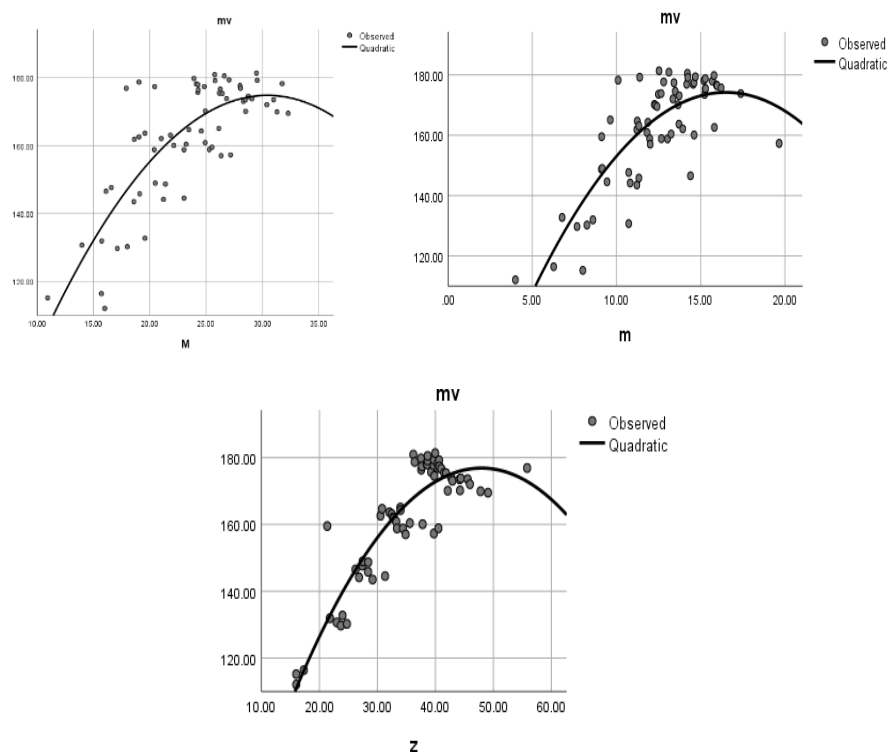


FIGURE 10 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (mv) with physical properties of 67 alkanes in Quadratic model

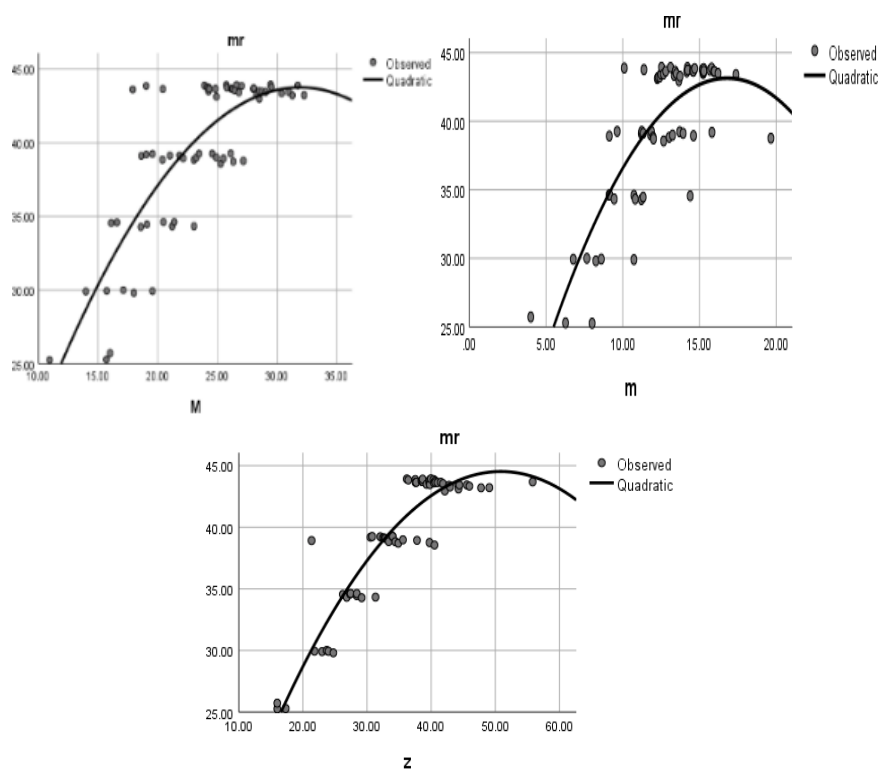


FIGURE 11 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (mr) with physical properties of 67 alkanes in Quadratic model

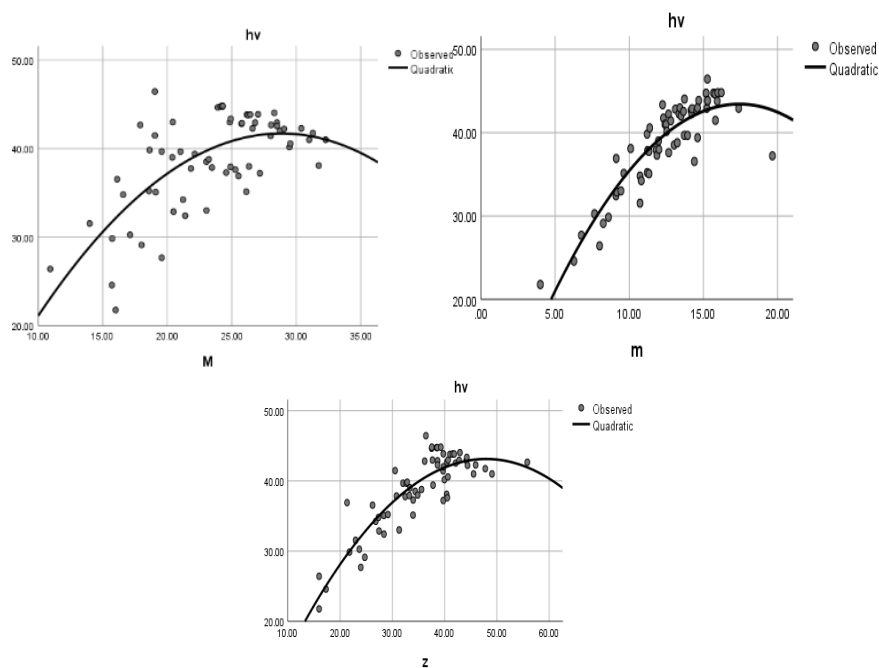


FIGURE 12 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (hv) with physical properties of 67 alkanes in Quadratic model

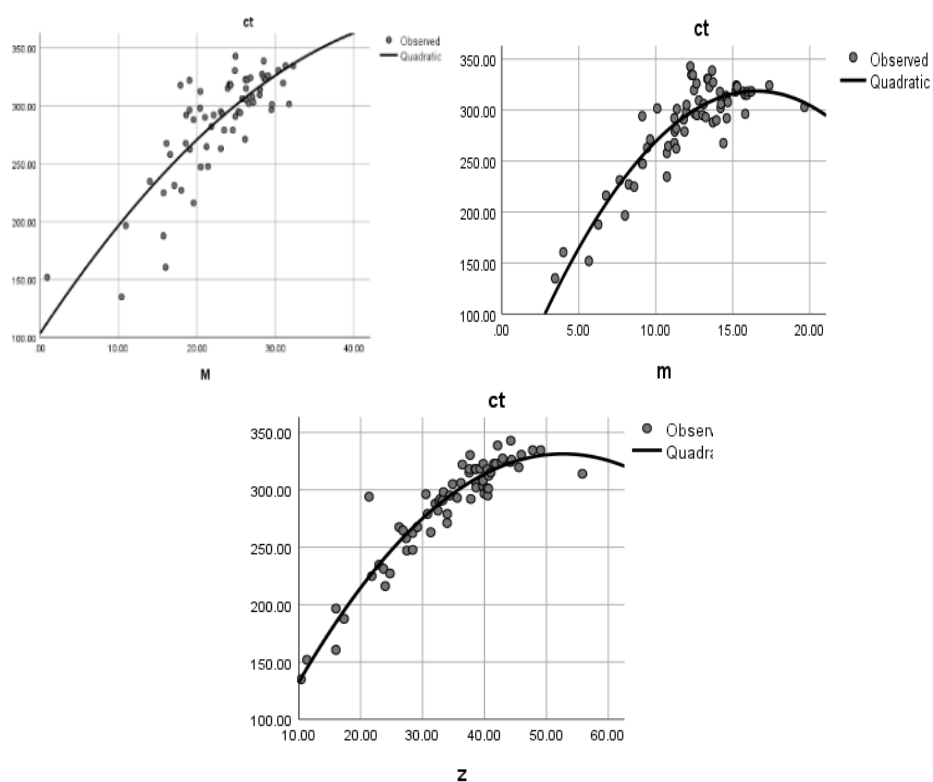


FIGURE 13 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (ct) with physical properties of 67 alkanes in Quadratic model

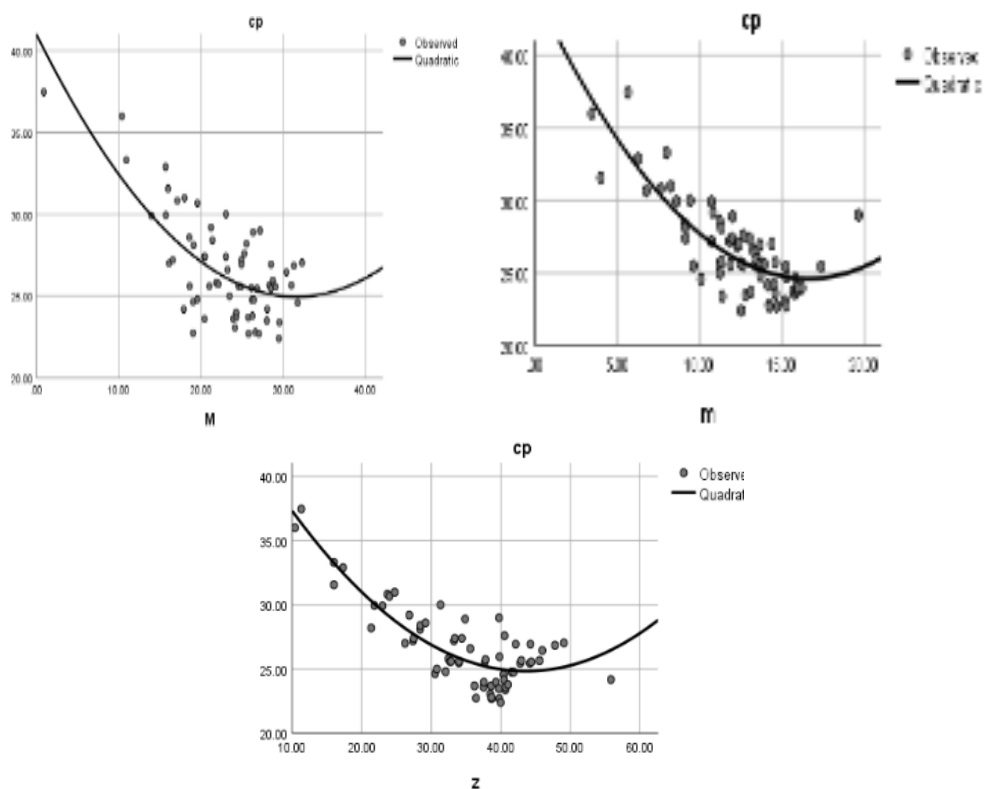


FIGURE 14 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (cp) with physical properties of 67 alkanes in Quadratic model

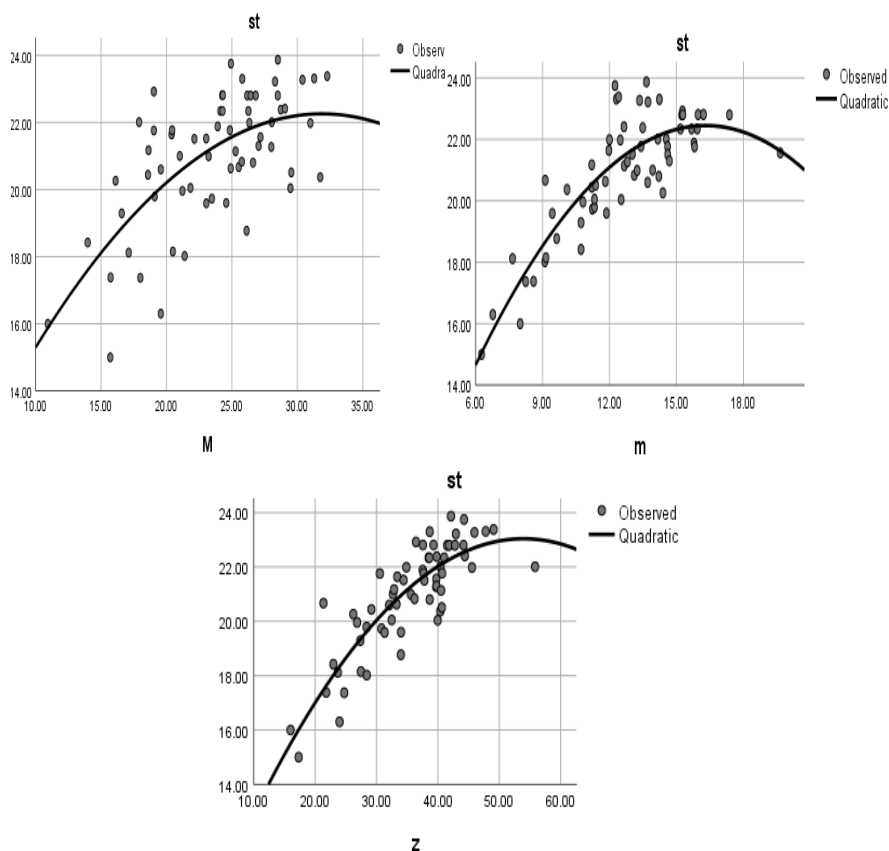


FIGURE 15 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (st) with physical properties of 67 alkanes in linear model

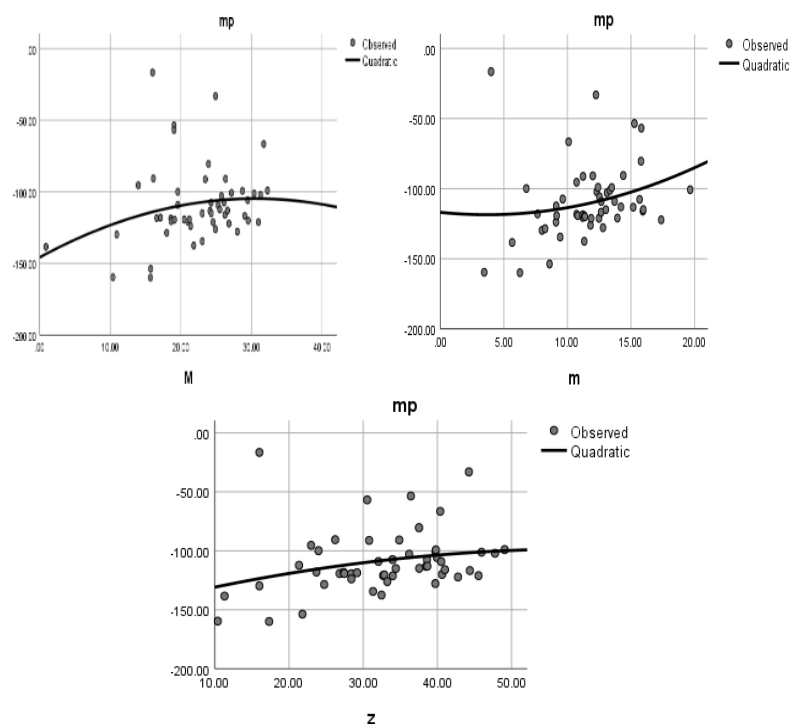


FIGURE 16 Correlation between Maximum Degree Energy, Minimum Degree Energy, and Second Zagreb Energy (mp) with physical properties of 67 alkanes in Quadratic model

Conclusion

The correlation coefficient (r) shows the eligibility level of the topological index as a tool to predict the physico-chemical properties of chemical compounds in the QSPR analysis. The results of QSPR studies reveals that the regression model is the most significant model to predict the physico-chemical properties of chemical compounds.

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Conflict of interest

There are no conflicts of interests.

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